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Transmission Zeros and High-Authority/Low-Authority Control of Flexible Space Structures

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Flexible space structures generally have very low levels of inherent damping, which makes the problem of controlling such systems quite challenging. For instance, unmodeled modes can easily be made unstable by spillover effects. One way to avoid these difficulties is to use a low-authority control loop to simply increase structural damping, typically to 5–10% of critical, and then use a more sophisticated high-authority control loop to achieve the desired control objectives. A very important design question that then arises is: what are the best choices of locations for the dampers used to implement the low-authority controller? This question is studied here, and it is shown that it is more important to increase the damping of the zeros of the damping-augmented structure than that of the poles. A simple algorithm is then derived for determining the damper locations that make the zeros as heavily damped as possible. Finally, the operation of this algorithm is illustrated by application to a simple cantilever beam example.

Introduction

THE very low damping typical of the vibration modes of flexible space structures (FSS) greatly complicates the problem of controlling such vehicles because if any of these modes are excited by the controller or disturbances, they will take a considerable time to come to rest. A technique that has been proposed to try to overcome this difficulty is that of high-authority/low-authority control (HAC/LAC).[1,5,20,22] In this approach, a LAC loop, acting through dampers placed on the structure, is used to augment the inherent damping of the vehicle, typically raising it to 5–10% of critical. The intention is to make it easier for the main HAC loop, acting through its own actuators, to give good closed-loop system performance as measured at the HAC sensor stations. The increase in damping provided by the LAC loop also helps to prevent any unmodeled modes being destabilized by spillover from the main control loop. The high-authority controller is usually chosen to be an optimal regulator, as many FSS control problems are readily expressible in an optimization form; an example is the problem of minimizing the rms surface deflection of a flexible antenna without using excessive amounts of propellant.

The LAC design problem considered in this paper is as follows: Given an optimal regulator HAC loop with specified sensor and actuator locations and a fixed number of dampers of given strengths, where should these dampers be placed on the structure to give the best overall system performance? Central to this analysis are the results recently derived in Ref. 11 concerning the optimal root loci[8] of FSS, which in turn are closely related to the generic properties[19] of the transmission zeros[6] of these systems. In particular, if the HAC loop is permitted to use fairly high gains (as is necessary if reasonably good performance is to be obtained), then the finite poles of the closed-loop system (FSS + LAC + HAC) will approach the zeros of (FSS + LAC). Typical LAC analyses in the past[1,22] have concentrated on positioning dampers so as to maximize the damping of the poles of (FSS + LAC), with no attention being paid to the zeros of this damping-augmented system. It can therefore be seen that placing dampers in this traditional way will not necessarily improve the performance of the overall closed-loop system (FSS + LAC + HAC) significantly. This question will be studied in the paper and a simple algorithm given, based on orthogonality to the HAC modal influence matrix, for determining damper locations that produce as much zeros damping as possible. The poles will always also be damped by this scheme. The converse, however, would not be true: a method designed to maximize the damping of the poles may leave the zeros totally undamped. Numerical results for a simple cantilever beam will be used to illustrate these results.

High-Authority/Low-Authority Control

LAC of FSS is generally implemented[1] by a set of relatively low-effort actuators applying forces or torques proportional to the outputs of a set of rate sensors that are compatible (physically collocated and coaxial) with them. Each sensor/actuator pair typically operates independently, introducing a damping force proportional to the local rate measurement. The physical implementation of LAC may be passive (e.g., by dashpots or dampers) or active (e.g., proof-mass actuators[2] or active members[3]); it may also be by distributed rather than discrete means (e.g., by piezoelectric layers[4]). In all cases, however, the role of LAC is to introduce moderate amounts of damping into the otherwise very lightly damped structure, rather than to achieve stringent pointing requirements.

HAC, on the other hand, is[5] typically a more ambitious modern control scheme based on state feedback[6] (it therefore requires an observer in any practical implementation). The actuators for this controller are capable of greater force/torque than those of the LAC loop; the sensors may feedback displacements instead of rates; each actuator will typically take measurements from all sensors; and sensors and actuators need not be collocated. A HAC controller is capable of much better closed-loop performance than a LAC controller for the nominal system; however, it is generally far more sensitive to perturbations in the open-loop dynamics. For instance, if a noncollocated sensor and actuator vibrate in phase in the nominal model but out of phase in the true structure, negative rate feedback between them will be nominally stabilizing but actually destabilizing. The collocated rate feedback implemented by the LAC loop, on the other hand, is guaranteed[7] always to be stabilizing, regardless of plant perturbations. Furthermore, LAC will stabilize any unmodeled vibration modes, whereas HAC can destabilize these by spillover effects.

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One of the prime motivations for using LAC is, therefore, to stabilize any HAC-induced instabilities that may arise as a result of moderate perturbations to the plant. LAC was consequently described in Ref. 5 as an outer loop, serving to stabilize the inner closed-loop system (FSS + HAC). However, LAC can equally well be regarded[20] as the inner control loop and HAC as the outer; this is the formulation that will be used here. When the problem is posed in this form, it is not necessarily important to simply maximize the damping ratios of the poles of (FSS + LAC). Rather, the LAC should be designed so that the overall closed-loop system (FSS + LAC + HAC) has good performance. The results obtained here on optimal damper positioning are based on this philosophy and will prove to be quite different from those typically given in the past.

To study the HAC/LAC design problem in detail, consider an n-mode linear model for the structural dynamics of an undamped, nongyroscopic, noncirculatory FSS with m compatible HAC sensor/actuator pairs. This can be written as

$$M\ddot{q} + Kq = Vu \tag{1a}$$

$$y = W_r\dot{q} + W_d q \tag{1b}$$

where q is the vector of generalized coordinates, u that of applied HAC actuator inputs, and y that of HAC sensor outputs. The mass and stiffness matrices of the structure satisfy $M = M^T > 0$ and $K = K^T \geq 0$, respectively, the control influence matrix V is of full column rank, and $(W_r \;\; W_d)$ is of full row rank. Note that a modal model for this structure is simply a special case of Eq. (1) with $q = \eta$, the modal amplitude vector, so the results that will be derived below apply equally well to $\{M, K\}$ or modal models.

Applying LAC to this system corresponds to adding a term of the form $C\dot{q}$ to the left-hand side of Eq. (1a), where

$$C = V_L D V_L^T \tag{2}$$

D is diagonal with $r < s$ nonzero entries, in as yet unknown locations, corresponding to the specified LAC damper strengths $\{d_i : i = 1, \ldots, r\}$, and V_L is the $(n \times s)$ influence matrix corresponding to all possible damper locations. The LAC design problem as formulated here then consists of choosing those damper locations (i.e., that ordering for the diagonal entries of D) that will improve the performance of the overall closed-loop system (FSS + LAC + HAC) as much as possible.

The particular type of HAC to be studied is the linear optimal regulator,[8] where the objective is to find the control input u that minimizes the quadratic cost functional

$$J = \int_0^\infty [y^T Q y + \rho u^T R u] \, dt \tag{3}$$

where the weighting matrices Q and R are positive definite symmetric and the scalar ρ is nonnegative. Optimal regulators are often used[9,10] for the control of FSS as the desired objectives, for instance, minimizing the rms surface deflection of a flexible antenna without using excessive amounts of propellant, are typically readily expressible in a quadratic optimization form. A necessary prelude to deriving good LAC damper placement criteria is thus to study the properties of linear optimal regulators when these are applied to flexible structures. This was the subject of a recent paper,[11] and the relevant optimal root loci results form the basis of the next section.

FSS Optimal Root Loci

The optimal root loci of a system provide a great deal of information on the performance attainable when applying a linear optimal regulator to it. They do this by displaying graphically how the closed-loop poles vary as the control weighting parameter ρ in Eq. (3) is taken from ∞ to 0. These root loci start at the open-loop poles or the mirror images about the imaginary axis of any unstable poles; some then terminate at finite left half-plane locations, precisely the transmission zeros[6] of the system (or their mirror images), while the remainder tend to infinity in various Butterworth configurations.[8] Beyond these simple facts, though, no generic properties can be stated for the optimal root loci of general linear systems. In fact, it is not even possible in general to say immediately what the number and orders of the Butterworth configurations will be for a given multi-input system.

However, such generic results can be derived for the special case of lightly damped flexible structures. The behavior of an optimal regulator HAC loop when applied to a general damper-augmented flexible structure can therefore be described rather fully in graphical terms. This will then allow simple guidelines to be drawn up concerning where dampers should be placed to improve the performance of (FSS + LAC + HAC) as much as possible over that of (FSS + HAC) alone.

To study the optimal root loci of (FSS + LAC), note first that this open-loop system is guaranteed to be stable; its natural frequencies will be denoted by $\{\omega_i\}$ and its damping ratios by $\{\zeta_i\}$. The regulated closed-loop poles obtained for extremely high control weighting ($\rho \to \infty$) will therefore just be the poles $\{-\zeta_i\omega_i \pm j\omega_i\sqrt{1-\zeta_i^2}\}$. A question of some importance is to quantify how these poles move as ρ is decreased (i.e., as the performance measure $y^T Q y$ starts to be of significance). Reference 11 studied these angles and rates of departure and proved, taking Q and R as identity matrices for simplicity, that the ith locus departs as

$$\left.\frac{\partial\lambda}{\partial k}\right|_{k=0} = -\frac{1}{2}\sigma_i^2 \cdot (1 + j\zeta_i) \tag{4}$$

where $k = \rho^{-1}$ and σ_i^2 is the ith modal cost[12] (the contribution of mode i to $y^T y$). Thus, Eq. (4) shows that the speed with which a closed-loop pole is moved as ρ decreases is proportional to the contribution of this mode to J. This clearly makes sense. The direction in which poles move is also of interest: it is, to first order, such that their absolute values (i.e., undamped natural frequencies[13]) are preserved. Of course, if the system were undamped, Eq. (4) would reduce to a purely real pole shift. Finally, note that the derivation of Eq. (4) in Ref. 11 assumed that the observability correlation coefficients[14] between mode i and all other modes were small. This illustrates the dynamic significance of these coefficients: the root loci for uncorrelated modes evolve, for large ρ, essentially as uncoupled single degree-of-freedom harmonic oscillators, while those for highly correlated modes interact significantly. As correlated modes must correspond to close natural frequencies (but not necessarily vice versa!), this conclusion makes sense graphically.

Quantifying closed-loop system behavior for low control weighting ($\rho \to 0$) is of at least as much importance as the $\rho \to \infty$ case. The object here is to evaluate not only the angles and rates of approach of certain of the loci to the transmission zeros of (FSS + LAC), but also the directions and rates (i.e., Butterworth configurations) at which the remaining loci tend to infinity. Taking the asymptotically infinite loci first, it was shown in Ref. 11 that (for compatible HAC sensors and actuators) there are always precisely m of these branches. Those that correspond to rate measurements are first-order Butterworth configurations, each giving a closed-loop pole on the negative real axis with magnitude proportional to $\rho^{-1/2}$; those corresponding to displacement measurements are second-order, each giving a pair of poles varying as $\rho^{-1/4}$ with asymptotes at $\pm\pi/4$ to the negative real axis. These results are considerably simpler than those that apply[15] for general state-space systems. As a final point, extensive computer simulations of lightly damped compatible flexible structures suggest that the loci that terminate at isolated transmission zeros always approach them roughly horizontally from the left, just as the departure from an isolated pole is approximately horizontally to the left. However, a formal proof of this property has not yet been found.

Summarizing, Fig. 1 shows the typical form of the optimal root loci for an undamped flexible structure with no rate measurements. Any HAC optimal regulator that is to be useful must allow high enough control gains that the closed-loop performance is significantly faster than the open loop. Therefore, it must correspond to quite a low value for ρ, so it can be seen that the overall response will depend more on the zeros of (FSS + LAC) than on the poles of this damping-augmented system. A technique will now be derived for calculating the locations at which a specified set of dampers should be placed so as to make these zeros as heavily damped as possible.

Damper Placement for Zeros Damping

Taking the Laplace transform of Eq. (1), augmented by the LAC damping term $C\dot{q}$, yields the frequency domain polynomial matrix representation[16]

$$P(s)q(s) = Vu(s) \qquad y(s) = W(s)q(s) \tag{5}$$

for the system (FSS + LAC), where $P(s) = s^2M + sC + K$ and $W(s) = sW_r + W_d$. Note that $P(s)$ is symmetric (i.e., Eq. (5) respects the special form of the equations of motion of structural dynamics). This contrasts with the first-order state-space representation $\dot{x} = Ax + Bu$, $y = Cx$ with $x = (\dot{q}^T\ q^T)^T$, where A does not preserve this useful symmetric structure.

If the HAC actuators and sensors have been positioned in such a way that Eq. (5) is completely controllable and observable (i.e., so that each mode can be both excited and sensed), then the transmission zeros of this system are those s_i for which the system matrix[6]

$$S(s) = \begin{bmatrix} P(s) & V \\ -W(s) & 0 \end{bmatrix} \tag{6}$$

loses rank. The zeros are equivalently those s_i that reduce the rank of the rational transfer matrix $T(s) = W(s)P(s)^{-1}V$, but the polynomial matrix $S(s)$ is generally far more convenient to deal with. In fact, it can be transformed to a canonical form from which the zeros are very readily computable.[17] This is based on the QR decomposition[18] of V, that is, the orthogonal Q and rectangular $R = (B^T\ 0)^T$ (B square and upper triangular) for which $V = QR$. Because the HAC sensors and actuators are taken to be compatible, this same transformation, which reduces Q^TV to upper triangular form, also reduces W_rQ and W_dQ to $(D_r\ 0)$ and $(D_d\ 0)$, respectively, where D_r and D_d are both square and $(D_r\ D_d)$ has full row rank. Thus, the system matrix $S(s)$ can be reduced by the orthogonal transformation $Q_s = \mathrm{diag}\,\{Q, I\}$ to

$$\bar{S}(s) = Q_s^TS(s)Q_s = \begin{bmatrix} Q^TP(s)Q & Q^TV \\ -W(s)Q & 0 \end{bmatrix} = \left[\begin{array}{cc|c} Q_1^TP(s)Q_1 & Q_1^TP(s)Q_2 & B \\ Q_2^TP(s)Q_1 & Q_2^TP(s)Q_2 & 0 \\ \hline -D(s) & 0 & 0 \end{array}\right] \tag{7}$$

where $Q = (Q_1\ Q_2)$ and $D(s) = sD_r + D_d$. By inspection then, the transmission zeros of the system are those s_i for which either the $(m \times m)$ $D(s)$ is singular (the sensor zeros) or the $(n-m)\times(n-m)$ $Q_2^TP(s)Q_2$ is singular (the structural zeros). It can be shown[19] that the $(n-m)$ conjugate pairs of structural zeros, which depend on the physical properties of the structure and the positions chosen for sensor/actuator pairs, always lie in the left half-plane. They have many additional properties if damping is taken to be modal; however, LAC damping does not generally take this form, so details will not be given here. (The interested reader is referred to Ref. 19.) The sensor zeros depend only on the way in which rate and displacement measurements go into the system outputs; these zeros generally lie on the negative real axis.

The effect of LAC on the zeros of (FSS + LAC) is thus to add the term $Q_2^TCQ_2$ to the polynomial matrix that defines the structural zeros. If our goal is to make these zeros as damped as possible, as argued in the last section, then a good rule-of-thumb is to try to find damper locations that make some norm of $Q_2^TCQ_2$ as large as possible. (The particular norm that is chosen is not particularly important.) To gain insight into this problem, consider a very simple implementation of HAC/LAC where local rate feedback is used at the HAC stations to generate the LAC damping. (The HAC/LAC separation is then notional rather than physical.) The LAC influence matrix V_L in $C = V_LDV_L^T$ then clearly satisfies $\mathrm{Im}(V_L) = \mathrm{Im}(V)$; thus, as $Q_2^TV = 0$ we must also have $Q_2^TV_L = 0$. An important consequence of this is that $Q_2^TCQ_2$ must be zero, so this arrangement does not introduce any damping into the zeros of (FSS + LAC). An alternative way to prove this is to note that this particular form of LAC is dynamically equivalent to state feedback. It therefore[6] cannot alter the zeros of the system (FSS + LAC), so these are just equal to the undamped zeros of the basic structure.

Thus, if we are to achieve a considerable amount of zeros damping, it is necessary to choose damper locations (i.e., place the nonzero diagonal elements of D) so that some norm of

$$Q_2^TCQ_2 = Q_2^T[V_LDV_L^T]Q_2 = [Q_2^TV_LD^{1/2}][Q_2^TV_LD^{1/2}]^T \tag{8}$$

is as great as possible. But $\mathrm{Im}(Q_2)$ is just the orthogonal complement[18] of V^T, so what is required is to choose D so as to make $V_LD^{1/2}$ "as orthogonal as possible" to V. Clearly then, LAC dampers should be placed at those locations that correspond to the columns of V_L that are most nearly orthogonal to V. The example given above of coincident HAC and LAC hardware can be seen to go entirely against this guideline. Requiring that V and $V_LD^{1/2}$ be as orthogonal as possible also makes good physical sense because, if we have a specified set of HAC sensor/actuator stations and then add dampers that merely affect the dynamics of the structure in roughly the same way $[\mathrm{Im}(V_LD^{1/2}) \approx \mathrm{Im}(V)]$, the overall response of the system will not be improved materially. It is more effective to put dampers at locations where they can "mop up" those dynamics that were not controlled adequately by the HAC loop.

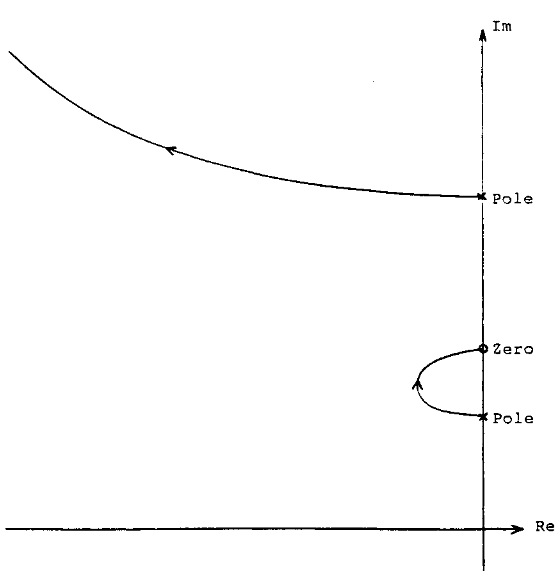

Fig. 1 Typical optimal root loci of FSS.

To reduce this guideline to an algorithm for damper placement, it is necessary to select a particular matrix norm of $Q_2^T C Q_2$ to be minimized. Now, any such norm must satisfy the consistency condition $\|XY\| \le \|X\| \cdot \|Y\|$ for any X and Y,[18] so Eq. (8) implies that $\|Q_2^T C Q_2\| \le \|Q_2^T V_L D^{1/2}\| \cdot \|[Q_2^T V_L D^{1/2}]^T\|$. Thus, if we choose a norm that satisfies $\|X^T\| = \|X\|$ for all X (e.g., the 2-norm or the Frobenius norm, but not the 1-norm or ∞-norm), we have that

$$\|Q_2^T C Q_2\| \le \|Q_2^T V_L D^{1/2}\|^2 \tag{9}$$

The Frobenius norm is much simpler to compute than the 2-norm, involving a sum of squares rather than a singular value decomposition,[18] so the damper placement philosophy becomes that of choosing so as to maximize $\|Q_2^T V_L D^{1/2}\|_F^2$. But this quantity is just $\Sigma_{j=1}^{s}$ [d_j · (Euclidean norm of column j of $Q_2^T V_L)^2$], so it will clearly be maximized if the largest d_i is positioned to go with the column of greatest Euclidean norm; the second largest d_i is paired with the second greatest column, etc. Of course, if all $r(<s)$ dampers are of equal strength, this procedure simply reduces to that of placing dampers at the locations that correspond to the r columns of $Q_2^T V_L$ with the greatest Euclidean norms.

Given the matrices V and V_L as defined in Eqs. (1) and (2), respectively, and the specified damper strengths $\{d_i : i = 1, \ldots, r\}$, this procedure can be summarized as the following formal algorithm.

Algorithm. Step 1: Find $V = QR$, $Q = (Q_1\ Q_2)$ orthogonal, $R = (B^T\ 0)^T$, the QR decomposition of V.

Step 2: Form the matrix $X = Q_2^T V_L$.

Step 3: The damper with the largest d_i value should be placed at the location corresponding to the column of X with the greatest Euclidean norm; the second largest damper should be placed at the location corresponding to the second greatest column norm, etc.

Step 4: $C = V_L D V_L^T$ is the desired damping matrix.

Note that this simple approach also gives insight into the question of how many dampers are actually needed. For, if only $q < r$ column norms of X are reasonably large, then using just q dampers will be nearly as effective as using all r.

Two final remarks concerning this algorithm are in order. The first is that the emphasis on the zeros of (FSS + LAC) does not mean that the poles of this system will be left undamped; they are clearly damped by any nonzero C. Thus, both extremes of the optimal root loci of (FSS + LAC) will be shifted away from the imaginary axis by C as constructed above, so the performance and robustness to spillover of (FSS + LAC + HAC) should be good even for quite low regulator gains. Second, the analysis given above has all been for compatible HAC sensors and actuators. However, the argument about high zeros damping being desirable carries over entirely without change to the noncollocated case; the difference is that the algebra is now much less straightforward, as the canonical form of Eq. (7) can no longer be used. This explains why the noncollocated sensor/actuator placement algorithm, recently independently derived by Maghami and Joshi,[23] and also based on maximizing the damping of the zeros, is more complicated than the method presented above.

Example

The results of the last section will now be illustrated by applying them to a simple flexible structure. Consider a truncated true modal model of order five for the transverse vibration of a uniform undamped cantilever beam of length 25 m, width 0.1 m, and depth 0.01 m, and constructed of aluminum (density 2.7×10^3 kg/m^3; Young's modulus 7.0×10^{10} N/m^2). A single HAC linear sensor/actuator pair is positioned at the beam tip, and we wish to determine which of five possible locations (at distances 12.5, 15.0, 17.5, 20.0, and 22.5 m from the built-in end) would be most suitable for a LAC damper.

The poles and zeros of the beam with no dampers are, of course, purely imaginary: their moduli are given in Table 1.

Table 1 Beam poles and zeros

Poles, rad/s	Zeros, rad/s
0.0827	
	0.3632
0.5182	
	1.1814
1.4510	
	2.4801
2.8433	
	4.2857
4.7002	

Table 2 Zeros damping ratios

	Damper location	
Zero	15.0 m	22.5 m
1	0.2318	0.0305
2	0.0058	0.0234
3	0.0194	0.0169
4	0.0140	0.0101
Mean	0.0677	0.0202

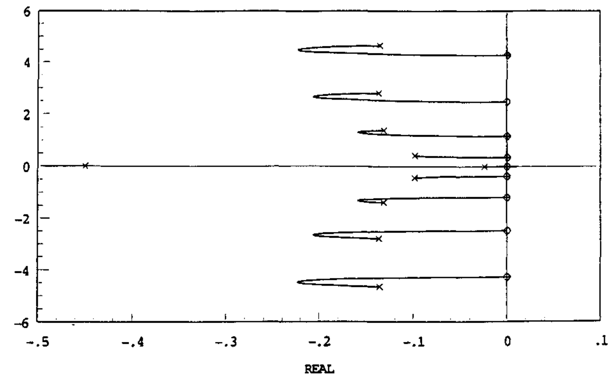

Fig. 2 Optimal root locus; tip damper.

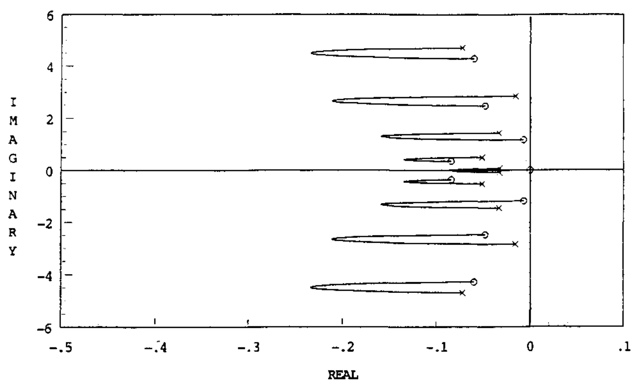

Fig. 3 Optimal root locus; damper at 15.0 m.

Note that the zeros interlace the poles, as is always the case[21] for a flexible structure with a single compatible (HAC) sensor/actuator pair, as here.

Applying the methods of the last section, it is found that the column of X corresponding to the damper position of 15.0 m has the greatest Euclidean norm (0.2824), whereas position 22.5 m has the lowest norm (0.2228). It is therefore expected that a damper placed at 15.0 m will be considerably more effective at introducing damping into the zeros of (FSS + LAC) than one placed at 22.5 m. Table 2 demonstrates that this is actually the case by displaying the damping ratios that are obtained for each zero, as well as the mean damping ratio, by placing a damper with $d = 5$ Ns/m at either of the two locations. As expected, the imaginary parts of each zero are not shifted by more than about 0.1% by this LAC. Furthermore, even though the damper position of 15.0 m was optimized for zeros damping, it still gives rise to an average pole damping ratio of 10.6%, which should be quite adequate for preventing spillover instabilities. As a graphical illustration of this point, Figs. 2 and 3 show the optimal root loci that are obtained for the beam when its damper is placed either at the tip (Fig. 2) or at 15.0 m (Fig. 3). In the first case, the poles of (FSS + LAC) are well-damped, as expected; however, the zeros remain purely imaginary. This fact, which was also expected, implies that the high-gain resistance of the closed-loop system to spillover instabilities will be unsatisfactory. By contrast, the damper location of 15.0 m selected using the new algorithm damps both poles and zeros well (i.e., the entire root locus is shifted to the left). This ensures resistance to spillover-induced problems for the entire range of HAC gains, as desired.

Conclusions

This paper has pointed out that the overall closed-loop response obtained when using HAC/LAC depends more on the zeros of the damping-augmented structure than on its poles. Consequently, the low-authority dampers should be placed in locations that will lead to the maximum possible amount of zeros damping. A simple algorithm to determine which damper positions will achieve this was derived in the paper and illustrated by application to a cantilever beam.

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